Cosmological Quantum Jump Dynamics

I. The Principle of Cosmic Energy Determinacy, Equations of Motion, and Jump Probabilities

Vladimir S. MASHKEVICH[1]

Physics Department
Queens College
The City University of New York
65-30 Kissena Boulevard
Flushing, New York 11367-1519

Abstract

The universe, as a closed system, is for all time in a state with a determinate value of energy, i.e., in an eigenstate of the Hamiltonian. That is the principle of cosmic energy determinacy. The Hamiltonian depends on cosmic time through metric. Therefore there are confluence and branch points of energy levels. At branch points, quantum jumps must happen to prevent the violation of energy determinacy. Thus quantum jumps are a reaction against the propensity of the universe dynamics to that violation. On the basis of this idea, an internally consistent quantum jump dynamics is developed.

[1]E-mail: Vladimir_Mashkevich@qc.edu

Introduction

Quantum jumps constitute the subject matter of the probabilistic aspect of quantum physics. A standard method for attacking the problem of quantum jumps has its basis in a unitary dynamics and the concept of decoherence, i.e., a transformation of a pure state into a mixed one. Such an approach can be traced back to the very advent of quantum theory [1]. There exists a voluminous literature on that approach. But it is impossible to construct an internally consistent dynamics of quantum jumps on the aforementioned basis, for unitary time evolution allows no decoherence.

In his lectures on gravitation [2], Feynman pointed out: "It is still possible that quantum theory does not absolutely guarantee that gravity *has* to be quantized... If there were some mechanism by which the phase evolution had a little bit of smearing in it, so it was not absolutely precise, then our amplitudes would become probabilities... But surely, if the phases did have this built-in smearing, there might be some consequences to be associated with this smearing. If one such consequence were to be the existence of gravitation itself, then there would be no quantum theory of gravitation... we should always keep in mind the possibility that quantum mechanics may fail, since it has certain difficulties with the philosophical prejudices that we have about measurement and observation."

Not quantizing gravity leads to semiclassical theory of gravitation. In the conventional treatment of semiclassical gravity, time evolution of a state vector Ψ of the matter of the universe is given by the Schrödinger equation $d\Psi/dt = -\mathrm{i}H_t\Psi$, where the Hamiltonian H_t depends on time through metric g. This time evolution, however, is unitary, which gives no way of constructing a consistent quantum jump dynamics.

But Feynman's considerations perceived in a broad sense imply that semiclassical gravity would be justified if it enabled us to construct an internally consistent quantum jump dynamics.

Along these lines, crucial is the following observation: Experience leads us to conclude that a closed system is for all time in a stationary state, i.e., in a state with a determinate value of energy. The universe is a closed system, so its energy should have a determinate value. That is the principle of cosmic energy determinacy. As the Hamiltonian H_t is time-dependent, the Schrödinger equation would have immediately violated the principle and should be abandoned.

The Hamiltonian is $H_t = \sum_l \varepsilon_l(t)P_l(t)$ where P_l is the projector for a level l. Denote a state vector by $|\,\rangle \equiv \Psi$ and the corresponding state projector by $P \equiv |\,\rangle\langle\,|$. Let the state belong to a level l, so that $H_t\Psi_t = \varepsilon_l(t)\Psi_t$. In this case, the state projector will be denoted by P^l. (Thus, subscript l designates a level, and superscript l a state belonging to that level.) Now $H_tP^l(t) = P^l(t)H_t = \varepsilon_l(t)P^l(t)$. For a degenerate level those conditions are insufficient. An equation of motion for P^l needs to be introduced. This is the one: $dP^l/dt = (dP_l/dt)P^l + P^l(dP_l/dt)$.

As the Hamiltonian depends on time, there are confluence and branch points of energy levels. At branch points, energy determinacy would be violated. It is quantum jumps that prevent the violation. Thus quantum jumps are a reaction against the propensity of the universe dynamics to the violation of energy determinacy.

As to the time t, it is universal cosmological time, the existence of which is implied by quantum jumps themselves: they give rise to a family of sets of simultaneous events—jumps of $\ddot{g} \equiv \partial^2 g/\partial t^2$.

The approach outlined above has been advanced in [3]; some results were obtained in [3-5]. The aim of this paper is to develop an internally consistent quantum jump dynamics.

1 The universe as a physical system

1.1 The Einstein equation and quantum jumps

We adopt the classical description of spacetime and quantum treatment of matter. The Einstein equation takes the form of

$$G - \Lambda g = 8\pi\kappa\tilde{T} \tag{1.1.1}$$

$$\tilde{T} = (\Psi, T\Psi) = \mathrm{Tr}\{PT\} \tag{1.1.2}$$

where G is the Einstein tensor, g is the metric, Λ is the cosmological constant, κ is the gravitational constant, T is the energy-momentum tensor operator, Ψ is a state vector, and

$$P = (\Psi, \cdot)\Psi = |\,\rangle\langle\,|\ , \quad |\,\rangle = \Psi, \quad \mathrm{Tr}\,P = 1 \tag{1.1.3}$$

is the corresponding state projector. We abandon the concept of decoherence, so it is reasonable to exploit P rather than Ψ.

A jump of P results in that of $\tilde{T}$. Write down (1.1.1) in components

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi\kappa\tilde{T}_{\mu\nu}, \quad \tilde{T}_{\mu\nu} = \mathrm{Tr}\{PT_{\mu\nu}\}, \quad \mu,\nu = 0,1,2,3 \tag{1.1.4}$$

The components G_{ij} $(i,j = 1,2,3)$ of the Einstein tensor involve the second time derivatives

$$\ddot{g}_{ij} \equiv g_{ij,00} \tag{1.1.5}$$

of the metric tensor components g_{ij} [6,7]. That makes it possible to retain the six equations

$$G_{ij} - \Lambda g_{ij} = 8\pi\kappa\tilde{T}_{ij} \tag{1.1.6}$$

unchanged. Jumps of the $\tilde{T}_{ij}$ will result in those of the $\ddot{g}_{ij}$, which is quite conceivable from the physical point of view: A jump of the force $\tilde{T}_{ij}$ results in a jump of the acceleration $\ddot{g}_{ij}$. As to the four equations

$$G_{0\mu} - \Lambda g_{0\mu} = 8\pi\kappa\tilde{T}_{0\mu} \tag{1.1.7}$$

the situation is completely different. The components $G_{0\mu}$ involve no second time derivatives of the $g_{\mu\nu}$; the only time derivatives of metric involved in $G_{0\mu}$ are the $\dot{g}_{ij}$. The latter should be continuous, not to mention the $g_{\mu\nu}$. The violation of the four equations (1.1.7) is intolerable and they must be extended, which has been done in [5].

1.2 Quantum-jump universal time

A quantum jump of the state projector P gives rise to a set of events—jumps of $\ddot{g}$. These events are, by definition, simultaneous, which allows for synchronizing clocks and thereby furnishing the universal time. It is natural to identify phenomenological cosmological time with the quantum-jump universal time. Thus cosmological time is defined on the level of fundamental physical laws—in contrast to the phenomenological approach in classical cosmology [8,9].

In special relativity, the concept of simultaneity in connection with quantum jumps makes no operationalistic sense, which complicates the incorporation of quantum jumps into special relativity. Taking gravity into account endows the concept with an operationalistic content.

1.3 Spacetime

The universal cosmological time gives rise to a family of sets of simultaneous events, thereby endowing spacetime with a fiber structure. The metric compatible with this structure, i.e., admitting the synchronization of clocks, is of the form [7]

$$ds^2 = g_{00}(dx^0)^2 + g_{ij}dx^i dx^j \tag{1.3.1}$$

or with

$$dt = \sqrt{g_{00}}dx^0, \quad t = t(x^0, \vec{x}) = \int g_{00}^{1/2}(x^0, \vec{x})dx^0, \quad \vec{x} = (x^1, x^2, x^3) \tag{1.3.2}$$

$$ds^2 = dt^2 + g_{ij}dx^i dx^j, \quad g_{ij} = g_{ij}(t, \vec{x}) \tag{1.3.3}$$

which relates to a synchronous reference frame. The latter, in its turn, implies the product spacetime manifold

$$M = M^4 = T \times S, \quad M \ni p = (t, s), \quad t \in T, \quad s \in S \tag{1.3.4}$$

The one-dimensional manifold T is the universal cosmological time, and the three-dimensional manifold S is a cosmological space. By (1.3.4), the tangent space M_p at a point $p \in M$ is

$$M_p = T_t \oplus S_s, \quad p = (t, s) \tag{1.3.5}$$

and, in view of (1.3.3),

$$T_t \perp S_s \tag{1.3.6}$$

Thus, quantum jumps give rise to the product spacetime (1.3.4) and a particular synchronous reference frame

$$p = (t, s) \leftrightarrow (t, \vec{x}) \tag{1.3.7}$$

The latter may be called cosmological reference frame and regarded as a canonical synchronous reference frame.

In the coordinate-free representation, the metric (1.3.3) reads

$$g = dt \otimes dt - h_t, \quad h_t \leftrightarrow h_{ij}(t, \vec{x})dx^i dx^j, \quad h_{ij} = -g_{ij} \tag{1.3.8}$$

in which h_t is a Riemannian metric tensor on S depending on t.

1.4 Matter

In what follows, quantities pertaining to matter are: the state projector P (1.1.3), the space part of the energy-momentum tensor operator

$$T_{\text{space}} \leftrightarrow T_{ij} \tag{1.4.1}$$

and the Hamiltonian

$$H_t = \int T_{00}(t, \vec{x})d\vec{x} \tag{1.4.2}$$

or in the coordinate-free representation

$$H_t = \int \mu(ds) T_{\text{time}}(t, s), \quad T_{\text{time}} \leftrightarrow T_{00} \tag{1.4.3}$$

2 Energy determinacy

2.1 The principle of cosmic energy determinacy

The cosmic energy determinacy principle reads: The universe is for all time in a state with a determinate value of energy, i.e., in an eigenstate of the Hamiltonian H_t. Note that a combination of the energy determinacy and a time-dependent Hamiltonian is analogous to the adiabatic potential method in quantum scattering theory.

2.2 Necessary conditions for the state projector

A necessary condition for the state projector P that follows from energy determinacy is

$$H_t P(t) = P(t) H_t = \varepsilon(t) P(t) \tag{2.2.1}$$

The spectral decomposition of the Hamiltonian is of the form

$$H_t = \sum_l \varepsilon_l(t) P_l(t) \tag{2.2.2}$$

where P_l is the projector for a level l with an energy ε_l. For a nondegenerate level

$$\mathrm{Tr}\ P_l = 1 \tag{2.2.3}$$

for a degenerate one

$$\mathrm{Tr}\ P_l \geq 2 \tag{2.2.4}$$

Let the state belong to a level l:

$$P = P^l, \quad \mathrm{Tr}\ P^l = 1 \tag{2.2.5}$$

Then (2.2.1) reduces to

$$P_{l'} P^l = P^l P_{l'} = \delta_{ll'} P^l \tag{2.2.6}$$

We call P_l a confining projector

$$P_l = P_{\mathrm{con}} \tag{2.2.7}$$

2.3 Extended confinement

For what follows, provision should be made for an extension of the confining projector

$$P_{\mathrm{con}} = \sum_{l \in L_{\mathrm{con}}} P_l \tag{2.3.1}$$

where L_{con} is a set of levels. The confinement condition for P takes the form of

$$P_{\mathrm{con}} P = P P_{\mathrm{con}} = P \tag{2.3.2}$$

3 Equations of motion: Confined dynamics

3.1 The state of the universe

The state of the universe is defined by metric, its time derivative, and the state projector:

$$\omega = (g, \dot{g}, P) \Leftrightarrow (h, \dot{h}, P) \leftrightarrow (h_{ij}, \dot{h}_{ij}, P) \Leftrightarrow (g_{ij}, \dot{g}_{ij}, P) \tag{3.1.1}$$

3.2 The equations for metric

The equations for metric are of the form

$$G_{ij} - \Lambda g_{ij} = 8\pi\kappa\tilde{T}_{ij} \tag{3.2.1}$$

or in the coordinate-free representation

$$G_{\text{space}} + \Lambda h = 8\pi\kappa\tilde{T}_{\text{space}} \tag{3.2.2}$$

We have

$$G_{\text{space}} = G_{\text{space}}[g, \dot{g}, \ddot{g}], \quad G_{\text{space}} \text{ is linear in } \ddot{g} \tag{3.2.3}$$

from here on we use square brackets to mean a functional. T_{space} does not depend on $\ddot{g}$; g and $\dot{g}$ are continuous in time.

3.3 The equation for the state projector: Confinement

In the case of

$$\text{Tr } P_{\text{con}} \geq 2 \tag{3.3.1}$$

(specifically when $P_{\text{con}} = P_l$ and the level l is degenerate), the confinement condition (2.3.2) is insufficient. An equation of motion for P should be introduced. From (2.3.2) follows

$$(dP_{\text{con}})P + P_{\text{con}}dP = (dP)P_{\text{con}} + PdP_{\text{con}} = dP \tag{3.3.2}$$

For any projector E

$$E(dE)E = 0 \tag{3.3.3}$$

holds. With equations (2.3.2) and (3.3.3) we can obtain a solution to (3.3.2) for dP of the form

$$dP = PdP_{\text{con}} + (dP_{\text{con}})P \tag{3.3.4}$$

Thus the equation of motion for P is

$$\frac{dP}{dt} = \left\{\frac{dP_{\text{con}}}{dt}, P\right\} \equiv \frac{dP_{\text{con}}}{dt}P + P\frac{dP_{\text{con}}}{dt} \tag{3.3.5}$$

We assume that the Hamiltonian

$$H = H[g] \tag{3.3.6}$$

does not depend on $\dot{g}$. Then in view of (2.2.2) and (2.3.1)

$$P_{\text{con}} = P_{\text{con}}[g] \tag{3.3.7}$$

so that

$$\frac{dP_{\text{con}}}{dt} = \frac{dP_{\text{con}}}{dt}[g, \dot{g}] \tag{3.3.8}$$

does not depend on $\ddot{g}$. Thus

$$\frac{dP}{dt} = \left\{ \frac{dP_{\text{con}}}{dt}[g, \dot{g}], P \right\} \tag{3.3.9}$$

so that dP/dt is given as a functional of the state ω (3.1.1) of the system.

Now

$$\dot{H} = \dot{H}[g, \dot{g}] \tag{3.3.10}$$

Thus H and $\dot{H}$ are continuous in time.

4 Vertices and tentative dynamics

4.1 Vertex

A vertex is a confluence or/and branch point of levels. We assume that the set of vertices has no accumulation points.

Consider an infinitesimal neighborhood of a vertex. For $t = t_{\text{vertex}} \equiv t_{\text{ver}}$ we have

$$H(t_{\text{ver}}) = H_{\text{ver}}(t_{\text{ver}}) + H_{\perp}(t_{\text{ver}}) \tag{4.1.1}$$

where

$$H_{\text{ver}}(t_{\text{ver}}) \equiv H_{\text{ver}} = \varepsilon_{\text{ver}} P_{\text{ver}} \tag{4.1.2}$$

For $t = t_{\text{ver}} \mp 0$ we have

$$H_{\text{ver}}(t_{\text{ver}} \mp 0) \equiv H^{\mp}_{\text{ver}}, \quad H^{+}_{\text{ver}} = H^{-}_{\text{ver}} = H_{\text{ver}} \tag{4.1.3}$$

and

$$\dot{H}_{\text{ver}}(t_{\text{ver}} \mp 0) \equiv \dot{H}^{\mp}_{\text{ver}}, \quad \dot{H}^{+}_{\text{ver}} = \dot{H}^{-}_{\text{ver}} = \dot{H}_{\text{ver}} \tag{4.1.4}$$

Now

$$H^{\mp}_{\text{ver}} = \sum_{l}^{1,n^{\mp}} \varepsilon^{\mp}_{l} P^{\mp}_{l} \tag{4.1.5}$$

where

$$\varepsilon^{\mp}_{l} = \varepsilon_{\text{ver}} \tag{4.1.6}$$

$$\sum_{l}^{1,n^{\mp}} P^{\mp}_{l} \equiv P^{\mp}_{\text{ver}} = P_{\text{ver}} \tag{4.1.7}$$

Next

$$\dot{H}^{\mp}_{\text{ver}} = \sum_{l}^{1,n^{\mp}} \dot{\varepsilon}^{\mp}_{l} P^{\mp}_{l} + \sum_{l}^{1,n^{\mp}} \varepsilon^{\mp}_{l} \dot{P}^{\mp}_{l} = \sum_{l}^{1,n^{\mp}} \dot{\varepsilon}^{\mp}_{l} P^{\mp}_{l} + \varepsilon_{\text{ver}} \dot{P}^{\mp}_{\text{ver}} \tag{4.1.8}$$

so that

$$\sum_{l}^{1,n^{+}} \dot{\varepsilon}^{+}_{l} P^{+}_{l} + \varepsilon_{\text{ver}} \dot{P}^{+}_{\text{ver}} = \sum_{l}^{1,n^{-}} \dot{\varepsilon}^{-}_{l} P^{-}_{l} + \varepsilon_{\text{ver}} \dot{P}^{-}_{\text{ver}} \tag{4.1.9}$$

Multiplying both sides of (4.1.9) from the left and from the right by P_{ver} and taking into account (4.1.7) we obtain

$$\sum_{l}^{1,n^+} \dot{\varepsilon}_l^+ (P_{\text{ver}} P_l P_{\text{ver}})^+ + \varepsilon_{\text{ver}} (P_{\text{ver}} \dot{P}_{\text{ver}} P_{\text{ver}})^+ = \sum_{l}^{1,n^-} \dot{\varepsilon}_l^- (P_{\text{ver}} P_l P_{\text{ver}})^- + \varepsilon_{\text{ver}} (P_{\text{ver}} \dot{P}_{\text{ver}} P_{\text{ver}})^- \quad (4.1.10)$$

In view of (3.3.3), it follows from (4.1.10) and (4.1.9) that

$$\sum_{l}^{1,n^+} \dot{\varepsilon}_l^+ P_l^+ = \sum_{l}^{1,n^-} \dot{\varepsilon}_l^- P_l^- \quad (4.1.11)$$

and

$$\dot{P}_{\text{ver}}^+ = \dot{P}_{\text{ver}}^- = \dot{P}_{\text{ver}} \quad (4.1.12)$$

4.2 Crossing and tangency

Consider equation (4.1.11). Let

$$\dot{\varepsilon}_l^+ = \dot{\varepsilon}_l^- \text{ and } P_l^+ = P_l^- \text{ for } l = 1, 2, \cdots, k, \quad k < \min\{n^-, n^+\} \quad \text{or} \quad k = n^- = n^+ \quad (4.2.1)$$

We call levels $l = 1, 2, \cdots, k$ crossing levels. For them

$$\sum_{l}^{1,k} P_l^+ = \sum_{l}^{1,k} P_l^- \equiv P_{\text{cross}} \quad (4.2.2)$$

Now

$$\sum_{l}^{k+1,n^+} \dot{\varepsilon}_l^+ P_l^+ = \sum_{l}^{k+1,n^-} \dot{\varepsilon}_l^- P_l^- \quad (4.2.3)$$

and

$$\sum_{l}^{k+1,n^+} P_l^+ = \sum_{l}^{k+1,n^-} P_l^- \equiv P_{\tan} \quad (4.2.4)$$

where, in view of what will follow, ‘tan’ stands for tangency, and

$$P_{\text{cross}} + P_{\tan} = P_{\text{ver}} \quad (4.2.5)$$

Let $k + 1 < \min\{n^-, n^+\}$ and, for the sake of definiteness, $n^+ \geq n^-$. From (4.2.3) follows

$$P_{l'}^+ \sum_{l}^{k+1,n-} \dot{\varepsilon}_l^- P_l^- P_{l''}^+ = 0, \quad l', l'' \in \{k+1, \cdots, n^+\}, \;\; l'' \neq l' \quad (4.2.6)$$

Substituting

$$P_{k+1}^- = P_{\tan} - \sum_{l}^{k+2,n^-} \quad (4.2.7)$$

into (4.2.6) we obtain

$$\sum_{l}^{k+2,n^-} (\dot{\varepsilon}_l^- - \dot{\varepsilon}_{k+1}^-) P_{l'}^+ P_l^- P_{l''}^+ = 0, \quad k + 2 \leq n^- \quad (4.2.8)$$

The number of the quantities $(\dot{\varepsilon}_l^- - \dot{\varepsilon}_{k+1}^-)$ is $[n^- - (k+1)]$, the number of equations is no less than

$$\frac{(n^- - k)^2 - (n^- - k)}{2} = \frac{n^- - k}{2}[n^- - (k+1)] \geq n^- - (k+1) \tag{4.2.9}$$

so that

$$\dot{\varepsilon}_l^- = \dot{\varepsilon}_{k+1}^-, \quad l = k+2, \cdots, n^- \tag{4.2.10}$$

Now

$$\sum_l^{k+1, n^+} \dot{\varepsilon}_l^+ P_l^+ = \dot{\varepsilon}_{k+1}^- P_{\tan} \tag{4.2.11}$$

whence

$$\dot{\varepsilon}_l^+ = \dot{\varepsilon}_{k+1}^-, \quad l = k+1, \cdots, n^+ \tag{4.2.12}$$

Thus

$$\dot{\varepsilon}_{k+1}^+ = \cdots = \dot{\varepsilon}_{n^+}^+ = \dot{\varepsilon}_{k+1}^- = \cdots = \dot{\varepsilon}_{n^-}^- \equiv \dot{\varepsilon}_{\tan} \tag{4.2.13}$$

which means tangency of the levels $l \geq k+1$.

4.3 Tentative vertex dynamics

Let at $t = t_{\text{ver}} - 0$ the state belong to a level $\bar{l} \in \{1, 2, \cdots, n^-\}$:

$$P(t_{\text{ver}} - 0) = P^{\bar{l}}(t_{\text{ver}} - 0) \tag{4.3.1}$$

Confined dynamics along with energy determinacy imply equations

$$P_{\text{con}}(t_{\text{ver}} - 0) = P_{\bar{l}}(t_{\text{ver}} - 0), \qquad P_{\text{con}}(t_{\text{ver}} + 0) = P_{\text{con}}(t_{\text{ver}}) = P_{\text{ver}} \tag{4.3.2}$$

That is the tentative vertex dynamics.

5 Quantum jumps and their probabilities: Actual dynamics

5.1 Energy determinacy and jumps

In the case of branching $(n^+ \geq 2)$, the tentative vertex dynamics would violate energy determinacy. To prevent the violation, quantum jumps must be introduced. Thus we have

$$P(t_{\text{ver}} - 0) = P^{\bar{l}}(t_{\text{ver}} - 0) \xrightarrow{\text{jump}} P(t_{\text{ver}}) = P^l(t_{\text{ver}} + 0) = \frac{P_l^+ P^{\bar{l}}(t_{\text{ver}} - 0) P_l^+}{\text{Tr}\{P_l^+ P^{\bar{l}}(t_{\text{ver}} - 0)\}}\,,$$
$$\bar{l} \in \{1, 2, \cdots, n^-\}, \quad l \in \{1, 2, \cdots, n^+\}, \quad n^+ \geq 2 \tag{5.1.1}$$

The state is continuous on the right. For the confining projector, we obtain

$$P_{\text{con}}(t_{\text{ver}} - 0) = P_{\bar{l}}(t_{\text{ver}} - 0) \xrightarrow{\text{jump}} P_{\text{con}}(t_{\text{ver}}) = P_l(t_{\text{ver}} + 0) \tag{5.1.2}$$

The set

$$\{P_l^+\}_{l=1}^{n^+} \tag{5.1.3}$$

is determined by

$$H(t_{\rm ver}+\tau) = H[g(t_{\rm ver}+\tau)] \quad \text{for } \tau \to 0 \tag{5.1.4}$$

i.e., by

$$H[g(t_{\rm ver}) + \dot{g}(t_{\rm ver})\tau] \quad \text{for } \tau \to 0 \tag{5.1.5}$$

which is independent of $\ddot{g}(t_{\rm ver})$ and, therefore, of $P(t_{\rm ver}-0)$.

5.2 Probabilities

The probability of the jump $\bar{l} \to l$ (5.1.1) is

$$w^{\bar{l}\to l} = \mathrm{Tr}\{P_l^+ P^{\bar{l}}(t_{\rm ver}-0)\} \tag{5.2.1}$$

In the case of crossing there is no jump.

5.3 Actual dynamics

The actual dynamics is the level-confined one:

$$P_{\rm con} = P_l \tag{5.3.1}$$

It involves quantum jumps. They are a reaction against the propensity of the universe dynamics to the violation of energy determinacy.

Acknowledgments

I would like to thank Alex A. Lisyansky for support and Stefan V. Mashkevich for helpful discussions.

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